When $y=b$, $\frac{dx}{dt}=0$, $\frac{dy}{dt}=0$, $\frac{d\theta}{dt}=0$; $\therefore C=2mgb$, and we have

$$\frac{dx^2}{dt^2}+\frac{dy^2}{dt^2}+k^2\frac{d\theta^2}{dt^2}=2g(b-y)\ldots(1).$$

Now $x=a-\theta r\sin\alpha\cos\beta$, $y=b-\theta r\sin\alpha\sin\beta$; then

$\frac{dx}{dt}=-r\sin\alpha\cos\beta\frac{d\theta}{dt}$, $\frac{dy}{dt}=-r\sin\alpha\sin\beta\frac{d\theta}{dt}$, $\frac{dx^2+dy^2}{dt^2}=r^2\sin^2\alpha\frac{d\theta^2}{dt^2}$; then

$$k^2\frac{d\theta^2}{dt^2}=\frac{k^2}{r^2\sin^2\alpha}.\frac{dx^2+dy^2}{dt^2}.$$

Substituting in (1),

$$\frac{dx^2+dy^2}{dt^2}.\frac{r^2\sin^2\alpha+k^2}{r^2\sin^2\alpha}=2g(b-y)=2g\theta r\sin\alpha\sin\beta\ldots(2).$$

But $dx^2+dy^2=ds^2$, in which $s=\theta r\sin\alpha$, or $ds=r\sin\alpha d\theta$; $\therefore$ (2) becomes

$$\frac{ds^2}{dt^2}=\frac{2r^2\sin^2\alpha}{r^2\sin^2\alpha+k^2}gs\sin\beta\ldots(3).$$

Taking the derivative of both members of (3) with respect to t, dividing by $\frac{ds}{dt}$, multiplying by dt and integrating twice, noticing that when $t=0$, $\frac{ds}{dt}=0$, and $s=0$, and finally putting $s=l=$the length of the trough,

$$s=\tfrac{1}{2}\frac{r^2\sin^2\alpha}{r^2\sin^2\alpha+k^2}gt^2,\ \text{whence}\ t=\frac{1}{\sin\alpha}\sqrt{\frac{(10\sin^2\alpha+4)l}{5g\sin\beta}},\ k^2\ \text{being}\ \tfrac{2}{5}r^2.$$

Also solved by G. B. M. Zerr and G. W. Greenwood.

DIOPHANTINE ANALYSIS.

136 Proposed by A H HOLMES, Brunswick, Maine.

In the equation in Diophantine Analysis: $2x^2+2x+1=\square=u^2$, show that u is always the sum of two squares.

Solution by L. E. NEWCOMB, Los Gatos, Cal.

$2x^2+2x+1=x^2+(x+1)^2\ldots(1)$.

Let $pq=x$ or $pq=x+1$ according as x is odd or even; then, for all integral values of x that satisfy (1), $\frac{1}{2}p^2-\frac{1}{2}q^2=x+1$ or $\frac{1}{2}p^2-\frac{1}{2}q^2=x$.

$\therefore p^2q^2+(\frac{1}{2}p^2-\frac{1}{2}q^2)^2=x^2+(x+1)^2=u^2$.

But $p^2q^2+(\frac{1}{2}p^2-\frac{1}{2}q^2)^2=(\frac{1}{2}p^2+\frac{1}{2}q^2)$. For p, substitute $m+n$, for q, $m-n$; then $(\frac{1}{2}p^2+\frac{1}{2}q^2)^2$ becomes $(m^2+n^2)^2$. Since $(m^2+n^2)^2=u^2$, $u=m^2+n^2$), the sum of two squares.